Conceptual Summary for E&M

This is meant to be a quick and dirty summary of the conceptual points presented during the second semester in AP Physics (that is, while we were studying E&M, minus the magnetism and induction sections). It is probably not definitive, but it hits most of the high points. Few equations are being used because this year's test is going to be a prose special, and formulas are not going to be as important as understanding the underlying physics.

Electrostatics:

--conductors have metallic bonding;

--the valence electrons in a conductor can move around freely;

--when external charge is brought close to an uncharged conductor, the valence electrons in the conductor will rearrange themselves so that the conductor will be attracted to the source;

--insulators have covalent bonding;

--the valence electrons in an insulator cannot move around freely outside their bonding couples;

--when external charge is brought close to an uncharged insulator, the electrons in the atoms of the insulator will either be attracted to or repulsed by the charge (depending upon whether the charge is positive or negative), spending more time on one side of the atom than the other—this effect (van der Waal's effect) will cause an internal polarization that will cause the insulator to be attracted to the external charge;

--bottom line: bring an external charge close to ANY electrically neutral structure, be it a conductor or insulator, and the structure will be attracted to the charge;

--if you have an external charge and want to charge a conductor with that same type of charge, touch the two together (in doing so, their electrical potentials, their voltages, will become the same);

--if you have an external charge and want to charge a conductor with the opposite charge, bring the charge close (but do not touch) so that like charge on the conductor's will move to the opposite side of the conductor (opposite to where the external charge is), then touch the side *opposite*—that will remove some of the like-charge leaving the conductor with a preponderance of the opposite charge;

--the unit for charge in the MKS system is *coulombs*;

--the force between two point charges (Coulomb's Law) is proportional to the magnitude of the two charges and inversely proportional to the distance between the two charges squared;

--the proportionality constant in Coulombs Law was the inverse of four-pi times the permittivity of free space, or $1/(4\pi\varepsilon_o)$;

--the direction of the force one point charge due to the presence of a second point charge is determined by whether the charges are like (repulsion) or unlike (attraction);

Electric fields:

--the magnitude of an electric field, evaluated at a point, gives you a measure of the *force per unit charge* available at that point due to a field-producing charge or charge configuration;

--that is, $\left|E_{\text{at a point}}\right| = \dfrac{\left|F_{\text{on q while at point}}\right|}{q}$;

--one set of units for an electric field is *newtons/coulomb*;

--electric fields are vectors, which means components often come into play when dealing with them;

--when the field-producing charge is a point-charge, the magnitude of the electric field, evaluated at a point some distance from the charge, is proportional to the magnitude of the charge and inversely proportional to the square of the distance between the charge and the point of interest;

--at a particular point, the direction of an electric field is defined as the direction a *positive test charge would accelerate* if put at the point and released;

--one approach for deriving an expression for the electric field function for an extended charge is to define a *differential point charge* dq, determine the *differential electric field* at the point of interest due to that bit of charge, break that *differential electric field* into components, capitalize on symmetry, then integrate to determine the total field due to all of the charges;

--when executing the operation outlined above, a linear charge density λ (coulombs/meter) is sometimes required;

Gauss's Law:

--a measure of the amount of field that passes through a surface is called flux;

--assume a flat surface: a surface area vector $\vec{A}$ (magnitude equal to the area of a surface; direction perpendicularly out from the surface's face) can be defined for the surface;

--if the surface alluded to above is placed in a vector field $\vec{E}$, a flux Φ_E will pass through the surface equal to the component of E perpendicular to the face times the area, or $\Phi_E = \vec{E} \bullet \vec{A}$;

--electric flux is a measure of the amount of electric field that passes through a surface;

--in terms of electric field lines, the more lines that pass through a given surface, net, the greater the electric flux through the surface;

--Gauss observed that if you place a *closed* surface in an electric field, the electric flux through that closed surface will be proportional to the charge enclosed inside the surface;

--Gauss's rational was that charge *outside* the closed surface would produce negative flux as a line moved into the surface and an equal amount of positive flux as the line exited the surface, so the only net flux would come from charge *inside* the surface;

--the proportionality constant in Gauss's law was the inverse of the permittivity of free space, or $1/\varepsilon_o$;

--Gauss's Law always works, even when there is no symmetry to the internal charge and the surface enclosing the charge;

--Gauss's Law can be used to derive electric field functions if you can find a surface that has one or more of the following characteristics:
a.) the magnitude of the electric field is constant everywhere on the surface;
b.) the flux is zero through the surface;
c.) the direction of the electric field vector and the area vector have the same relationship at every point on the surface;

--for point charge configurations or spherical geometries (balls of charge, etc.), spherical symmetry works for Gauss's Law;

--for extruded charge configurations like wires, cylindrical symmetry works for Gauss's Law;

--for sheets of charge, plugs work (the ends have non-zero flux quantities and the walls have zero flux quantities);

--the left side of Gauss's Law ($\int_S \vec{E} \bullet d\vec{A}$) in spherical situations, assuming symmetry is exploited, is always equal to the electric field magnitude E times the surface area of the Gaussian sphere, or $E(4\pi r^2)$.

--the left side of Gauss's Law ($\int_S \vec{E} \bullet d\vec{A}$) in cylidrical situations, assuming symmetry is exploited, is always equal to the electric field magnitude E times the surface area of the Gaussian cylinder of length L (without the end-caps), or $E(2\pi rL)$.

--the hard part of Gauss's Law is calculating the right side of the equation, the $\frac{q_{encl}}{\varepsilon_o}$ part;

--a volume charge density ρ or a surface charge density σ is often required to determine the charge enclosed in an imaginary Gaussian surface;

--as derived by Gauss's Law, the electric field outside an infinitely large conducting sheet is equal to $\frac{\sigma}{\varepsilon_o}$, where σ is the charge density associated with the sheet;

--as derived by Gauss's Law, the electric field outside an infinitely large insulating sheet is equal to $\frac{\sigma}{2\varepsilon_o}$, where σ is the charge density associated with the sheet;

--it is important to notice that σ means something different in the case of an insulator versus a conductor;

--for a conductor, σ tells you how much charge is on the surface of one side of the sheet, per unit area--(example: you give me an area, σA gives you how much charge is on the sheet in that area);

--for an insulator, σ tells you how much charge is shot all the way through the insulator, per unit area--(example: you give me an area, σA gives you how much charge exists throughout the volume behind the area);

Electric Potentials and energy considerations:

--*absolute electric potential* is defined as the amount of potential energy per unit charge available ($V_{\text{at a point}} = \frac{U_{\text{of q eval at point}}}{q}$) at a point in an electric field;

--the units of the absolute electric potential is joules/coulomb, or the volt;

--absolute electric potentials only exist for electric force fields that are conservative (static charges produce these);

--there is a direct parallel between the math associated with potential energy functions and energy considerations, and electric potentials and energy considerations, E&M style;

--the *work per unit charge* done by an electric field as a charge moves from one point to another is equal to minus the change in the absolute electric potential between the two points (that is $\frac{W}{q} = -\Delta V$);

--put differently, the work done on a charge as it moves from one point to another in an electric field will equal the charge (sign include) time the voltage change between the points, or $W = -(\pm q)\Delta V$;

--as work is force dotted into displacement, *work per unit charge* done is *force per unit charge* (or the electric field) dotted into displacement, or $\vec{E} \bullet \vec{d}$ (this assumes a constant $\vec{E}$ and constant $\vec{d}$ and non-varying angle between the two vectors);

--in a situation in which the electric field is varying, or the path somehow varies relative to the direction of the electric field, the differential *work per unit charge* over a differential displacement will be the electric field dotted into the differential displacement $d\vec{r}$ (or $dx\hat{i}$), that being $\vec{E} \bullet d\vec{r}$, and the net *work per unit charge* over the macroscopic displacement will be the sum of all of those differential quantities, or $\int \vec{E} \bullet d\vec{r}$;

--combining the *work per unit charge* relationships, apparently the relationship between the electric field and the electric potential field associated with a field-producing charge configuration is $\Delta V = -\int \vec{E} \bullet d\vec{r}$;

--if you know an electric field function and you want its electric potential function, in other words, if you sum all the differential $-\vec{E} \bullet d\vec{r}$'s between where the field is zero and an arbitrary point, you will get a function for V(r);

--putting the above statement a little differently, $V(r) = -\int_{\text{zero point}}^{r} \vec{E} \bullet d\vec{r}$;

--looking at that relationship a little differently, apparently minus the electric potential field is the spatial antiderivative of the electric field, or the electric field function is minus the derivative to the electric potential function;

--putting the above statement a little differently, $\vec{E}(r) = -\frac{\partial V}{\partial r}\hat{r}$, where the partial derivative is meant to denote a spatial derivative of the electric potential in the radial direction holding all other variables constant;

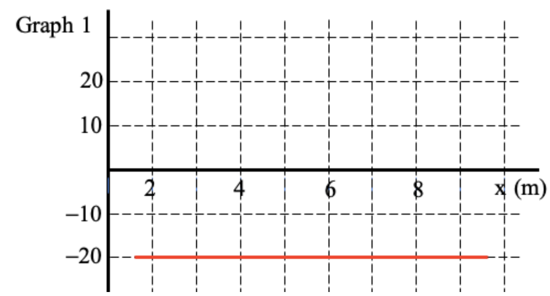
--translating this into cartesian geometry, we get, $\vec{E}(x) = -\frac{\partial V}{\partial x}\hat{i}$, where the partial derivative is meant to denote a spatial derivative of the electric potential in the x-direction holding all other variables constant;

--from this, another acceptable unit for electric field in the MKS system is *volts per meter*;

--for fields that have multiple dimensions, each component has a derivative associated with it so that in its most general form, $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$;

--the shortcut notation for this uses a del operator and looks like $\vec{E} = -\vec{\nabla}V$;

--the point is that at a given point in space, an electric field is related to (minus) the RATE its associated electric potential field CHANGES spatially;



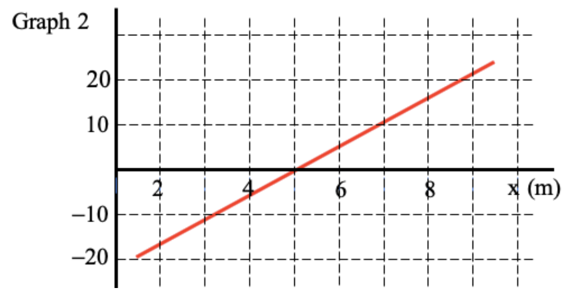
--if graph 1 is an *electric potential versus position* graph, what can you say about the region (solution below);

--assume graph 1 is "V vs x": the electric potential is negative and the same everywhere (-20 volts); as $\vec{E}(x)$ is minus the slope of V (that is, $\vec{E}(x) = -\frac{\partial V}{\partial x}\hat{i}$), the electric field in the region is zero;

--if graph 1 is an *electric field versus position* graph, what can you say about the region (solution below);

--assume graph 1 is "$\vec{E}(x)$ vs x": the electric field is negative and constant everywhere (-20 nt/C); as the *change of V* is minus the area under the curve (that is, $\Delta V = -\int \vec{E} \bullet d\vec{r}$), the voltage must *change uniformly* throughout in a positive sense (it must be getting bigger as x increases). In other words, its graph must have the general form of graph 2.

--if graph 2 is an *electric potential versus position* graph, what can you say about the region (solution below);



--assume graph 2 is “V vs x”: the electric potential is increasing at a constant rate with a positive slope, going from a negative voltage, through zero voltage at x = 5 and into the positive region thereafter; as $\vec{E}(x)$ is minus the slope of V (that is, $\vec{E}(x) = -\frac{\partial V}{\partial x}\hat{i}$), the electric field in the region is negative and constant (cause that’s what minus the slope is);

--if graph 2 is an *electric field versus position* graph, what can you say about the region (solution below);

--assume graph 2 is “$\vec{E}(x)$ vs x”: the electric field is negative near the origin with a large magnitude getting smaller at it approaches x = 5 where it is zero, then it changes directions becoming positive and increasing in magnitude as it proceeds in the +x-direction; as the *change of V* is minus the area under the curve (that is, $\Delta V = -\int \vec{E} \bullet d\vec{r}$), the voltage must change quite a bit in the region near the origin, changing less and less as it nears x = 5 where it changes not at all, then begins to change more and more as it continues on in the +x-direction. If we were looking at equipotential lines, they would be quite jammed up toward the origin, spreading out on either side of x = 5, then becoming increasingly more and more jammed up as you moved along the +x-axis. Lastly (and this is a bit over and above, but what the heh), as V is minus the anti-derivative of E, and as E is a positive linear function, you would expect V to be a negative quadratic (or downward parabolic). This last point is a little farther than you’d probably be expected to go, though.

--there are no defined electrical potential-energy functions in the same sense that there is a potential energy function for, say, a spring;

--the potential energy of a charge in an electric field will be U = qV, where the sign of the charge must be included (that is, positive charges have positive potential energies and negative charges have negative potential energies);

--as with all situation in which energy considerations can be used, conservation of energy is applicable with $\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$;

Capacitance:

--a capacitor is a circuit element made up of two plates that are electrically insulated from one another, and that have space (either filled with an insulator or not) between the plates;

--in an electrical circuit, charge accumulates on one plate electrostatically repulsing like charge off the other plate leaving it with the opposite charge—that means the plates will hold equal amounts of opposite charge;

--it has been experimentally observed that the voltage across a capacitor is proportional to the charge on one plate; the proportionality constant is called the capacitor's *capacitance*;

--the units for capacitance are *coulombs per volt*, or the farad;

--conceptually, what the capacitance tells you is *how much charge the device can hold, per unit volt*;

--in a DC setting, a capacitor stores energy in the electric field between its plates;

--a dielectric is an insulating material that can be put between the plates of a capacitor;

--the dielectric constant κ is the proportionality constant between the capacitance of a cap with a dielectric and the capacitance of the same cap without a dielectric (that is, $C_{\text{with diel}} = \kappa C_{\text{w/o diel}}$);

--some books use the symbol ε_d for the dielectric constant instead of κ;

--due to the van der Waal effect, a dielectric drops the effective voltage across the plates increasing the capacitor's capacitance (remember, C = Q/V);

--you can increase the capacitance of a capacitor in one of three ways:
a.) increasing the plate area will allow for more charge to be stored on a plate per volt;
b.) bringing the plates closer together will allow more charge to be repulsed off the opposite plate thereby increasing the charge per volt;
c.) place a dielectric between the plates increases the capacitor's capacitance;

--capacitors in series act like resistors in parallel;

--capacitors in parallel act like resistors in series;

--the dielectric constant was needed to allow q_{encl} in the Gauss's Law equation to be solely associated with the charge on the cap plate (ignoring the induced charge on the insulating dielectric)—that equation became $\int_S \vec{E} \bullet d\vec{A} = \frac{q_{encl}}{\kappa \varepsilon_o}$;

--to derive the capacitance of a capacitor in terms of physical parameters (that is, to determine $\frac{Q_{\text{on one plate}}}{\Delta V}$), you need to derive an expression for the voltage difference ΔV between the plates of the cap in terms of the charge on one plate; to do that:

a.) for the cap's geometry, use Gauss's Law to determine the electric field between the cap's plates;

b.) use the relationship between electric fields and electrical potential differences to determine the voltage difference across the plates (that is, use $\Delta V = \int \vec{E} \bullet d\vec{r}$);

c.) determine capacitance using $C = \frac{Q}{\Delta V}$;

--capacitors are used whenever one needs a quick infusion of charge (example: powering a flashbulb in a camera);

--in an RC circuit in which the capacitor is uncharged, the capacitor will initially act like a short with all the voltage drop across the elements happening across the resistor—that means the initial current for such a situation will be V/R;

--the time constant for a capacitor/resistor circuit is RC;

--in a RC circuit in which the capacitor is initially uncharged:

a.) it will take one time constant (RC) for the cap to charge to 63% of its maximum;

b.) because there is no charge initially on the capacitor's plates, there will be no voltage across the cap and all the voltage drop will be across the resistor; if the battery voltage is V, all that voltage will be across the resistor and the current through the circuit will initially be V/R.

c.) after a long time, current will go to zero because the cap will have completely charged up and the battery will no longer be able to force charge on the cap plates;

d.) putting "c" differently, after a long time, current will go to zero as all the battery's voltage drop will happen across the charge cap with no voltage drop across the resistor, and as current through a resistor is proportional to the voltage across the resistor, that means no current in the circuit;

e.) after a long time with all the battery voltage across the cap, the charge on the cap will be $Q_{max} = CV_{battery}$ (this from the definition of capacitance $C = \frac{Q}{V}$)

f.) it will take one time constant for the current to drop out 63% to 37% of its initial value (which was its maximum);

--in a RC circuit in which the capacitor is initially charged and there is no battery, just a resistor:

a.) the cap will discharge through the resistor, acting like a non-linear power supply for the circuit;

b.) after one time constant (RC) the cap will discharge 63% of its maximum charge down to 37%;

c.) because the initial amount of charge available for discharge is maximum, the current will begin as large as it will be and will diminish 63% worth down to 37% of its maximum in one time constant (RC).

d.) after a long time, current will go to zero because there will be no more charge on the cap;

Circuits:

--DC power in a circuit is provided by a DC power supply or battery, both of which have a high voltage terminal and a low voltage terminal;

--the voltage difference across the terminals of a DC power source generate an electric field in the wires connected to the source this motivate charge to flow through the circuit;

--a voltmeter is the device used to measure the voltage difference across an element (like a power supply)—it is placed across an element;

--voltmeters are polar, which is to say they have a high voltage terminal and a low voltage terminal;

--as charge carriers are accelerated by the electric field, flowing through the circuit, they crashes into the atomic structure of the wire giving up their kinetic energy in the process—this energy loss is absorbed by the system either by having the wire's atoms throw their valence electrons into higher energy levels (whereupon they cascade back down to the ground state, giving off photons . . . this is how light is made) or making the molecular structure vibrate more (this shows itself as heat);

--electrons move very slowly in an electric circuit, but the electric field sets itself up a near the speed of light;

--current is a measure of the amount of charge that passes a point per unit time;

--the units for current is coulomb/second, aka the ampere (or just amp);

--in a circuit, a branch is section of the circuit in which the current is the same throughout;

--in a circuit, a junction is called a *node*;

--an ammeter is the device used to measure the current through a circuit; it is placed in line in the branch in which the current value is desired;

--a resistor is a circuit element that does two things: it limits the amount of current that flows in a branch (a big resistor means a small current, etc.) and it converts electrical energy into some other form of energy (like an incandescent light bulb whose filament takes electrical current and uses it to generate light, or the toaster that takes electrical energy and turns it into heat).

--Ohm's Law states the voltage across a resistor will be proportional to the current through a resistor (circuit elements that act this way are called *ohmic*);

--the proportionality constant that relates the voltage across and current through a resistor is called the resistor's *resistance R*;

--the units of resistance, volts/amp, is named *ohm* and its symbol is Ω;

--resistors in series connect to one another in one place only;

--there are no nodes (no junctions) interior to a series combination;

--resistors in series have an equivalent resistance equal to the sum of the resistors in the combination;

--if you add a resistor to a series combination, the equivalent resistance goes up and the current into the branch goes down (for a given voltage);

--the equivalent resistance of a series combination will always be larger than the largest resistor in the combination;

--resistors in parallel connect to one another in two places;

--there are nodes (no junctions) interior to a parallel combination;

--resistors in parallel have an equivalent resistance equal to the inverse of the sum of the inverse resistances in the combination;

--if you add a resistor to a parallel combination, the equivalent resistance goes down and the current into the combination goes up (for a given voltage);

--if you add a resistor to a parallel combination, assuming the voltage across the combination doesn't change (that is, using an ideal battery), the current through each of the original resistors will not change (same voltage across each) but the current drawn from the battery will increase to accommodate the needs of the new resistor. From the battery's perspective, this need for more current looks like the effective resistance had diminished (lower the effective resistance and the current goes up), which is the conceptual justification for the observation made directly above.

--the equivalent resistance of a parallel combination will always be smaller than the smallest resistor in the combination;

--in analyzing a circuit *by the seat of your pants*, is possible to track the voltage changes around a path to determine the voltage difference between two points in a circuit;

--in analyzing a simple, single battery circuit *by the seat of your pants*, it is sometimes possible to determine the equivalent resistance of the resistors in the circuit and put that value across the battery voltage to determine the current drawn from the battery;

--in analyzing a circuit *by the seat of your pants*, start where you know the most information and work outward;

--a more formal way to analyze circuits is Kirchoff's Laws;

--Kirchoff's First Law is: the sum of the currents into a node equals the sum of the currents out of that node;

--Kirchoff's Second Law is: the sum of the voltage changes around a closed loop must equal zero;

--a loop is defined as any path that starts in a circuit and comes back to the start point;

--with an ideal battery in parallel with a parallel combination of resistors, adding a resistor to the combination will do nothing to the current through any of the parallel resistors (the voltage will not have changed across them) but the current drawn from the battery will have increased;

--with a real, non-ideal battery in parallel with a parallel combination of resistors, adding a resistor to the combination will drop the current through each of the parallel resistors; this is because internal resistance in the battery produces a current-dependent voltage drop internal to the battery, so the terminal voltage, which is equal to the battery's EMF minus that internal voltage drop, will diminish when the current increases due to the addition of that resistor.